



Hill, PR., Canagarajah, CN., & Bull, DR. (2001). Statistical wavelet subband modelling for texture classification. In *ICIP 2001* (Vol. 1, pp. 165 - 168). Institute of Electrical and Electronics Engineers (IEEE).
<https://doi.org/10.1109/ICIP.2001.958979>

Peer reviewed version

Link to published version (if available):
[10.1109/ICIP.2001.958979](https://doi.org/10.1109/ICIP.2001.958979)

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STATISTICAL WAVELET SUBBAND MODELLING FOR TEXTURE CLASSIFICATION

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ABSTRACT

Simple wavelet and wavelet packet transforms have often been used for texture characterisation through the analysis of spatial-frequency content. However, most previous methods make no use of any statistical analysis of the transforms' subbands. A novel method is now presented for modelling the multivariate distributions of subband coefficients by considering spatially related coefficients. The Bhattacharya and divergence metrics are then used to produce an improved texture classification method for the application to content based image retrieval.

1. INTRODUCTION

The Discrete Wavelet Transform (DWT) has been the focus of much research over the last decade. This has mainly been due to its use within efficient coding algorithms [1]. However, recently the DWT has been effectively used for texture analysis and representation. Gabor filters have been the most widely used method for texture analysis due to their excellent results and biological plausibility. Although potentially not as flexible, it has been shown [2] that texture classification using wavelets can perform as well as Gabor filters but with increased computational efficiency.

Many different methods have been proposed for extracting texture features from wavelet decompositions. Simple energy measures extracted from each subband within a dyadic decomposition can be used very effectively to classify textures [2]. Although often effective, average energy features make no account of the spatial variation of coefficients. Analysis and modelling of subband statistics has been common in image coding and noise removal applications [3]. However, analysis of subband statistics for classification has been less common. Choi *et al.* [4] have produced effective models of wavelet, and complex wavelet subband statistics using mixed Gaussians and hidden Markov models for image characterisation. However, these models are based on natural images rather than texture images.

In addition, Simoncelli and Adelson have produced a statistical analysis of complex wavelet coefficients [3]. However this method was aimed at texture synthesis and would be excessively complex for the simpler representations required for texture classification.

2. ANALYSIS STRUCTURE FOR WAVELET FEATURE VARIATIONS

A common approach to model the statistical variation of subband energies for texture classification is to use many example images of each texture. The variations of features extracted from each image can then be used by classifiers such as Neural networks [5] or minimum Mahalanobis distance classifiers [6] to produce a trained classification system. Within a content based image retrieval scenario this is of course impossible as there is usually only one image of each type in the database. One solution to this problem is to tile each image into small (e.g. 16×16) pixel regions [5, 7] and extract features from a wavelet transform performed on each region. In this way numerous feature vectors can be extracted from each training texture.

Instead of performing a wavelet decomposition on small tiled image regions and analysing the feature distributions, an alternative scheme can be constructed from a wavelet decomposition of the entire image. In the new scheme, the original image is tiled into 8×8 imaginary square regions (see figure 1). For each tiled region, the average of all the spatially related subband coefficients in each subband is calculated using the L_2 norm. This means that for all the lowest frequency subbands the energy is that of the single spatially related coefficient. For the other scales the energy is averaged over coefficient regions of 2×2 (middle scale) and 4×4 (highest frequency subbands) (see figure 1). A feature vector of length 10 is therefore constructed for each imaginary tiled region. The justification of using this method was that the variation of a repeating texture pattern (texel) should be well represented by analysing these small regions.

2.1. Texture Classification Experiments

Within all the subsequent experiments a simple dyadic wavelet decomposition is used, using a typical 9-7 linear phase wavelet pair. There are 10 resulting subbands as depicted in figure 3. A simple set of texture classification experiments was conducted to test the developed methods and metrics. All 112 individual Brodatz album textures were used in the experiment. 25 versions of each texture (of dimension 128×128 pixels) were used. One version of each texture class was used for training and 24 versions for test-

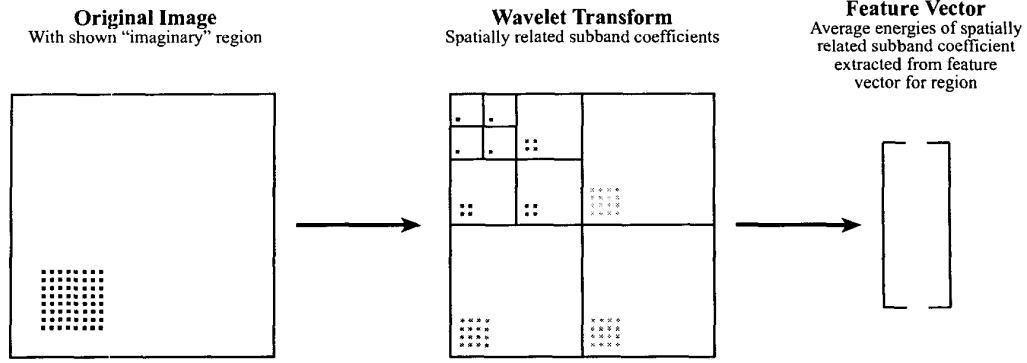


Fig. 1. Structure for extraction of statistics from wavelet decomposition

ing making 2688 tests in all. Using one training texture gives a better analogy to a similarity search within a content based image retrieval system.

Each test and training texture image is decomposed using a wavelet transform described above. The image is partitioned into $256 \times 8 \times 8$ "imaginary regions". A feature vector of length 10 is extracted from each region as described above. A mean feature vector and covariance matrix is calculated from these 256 feature vectors. A minimum distance classifier is used utilising the mean feature vector and covariance of the test and training images to classify each test texture image. i.e. the distance metrics described below are used to compare the test texture image with each training texture class.

3. BHATTACHARYA AND DIVERGENCE DISTANCE METRICS

The Mahalanobis distance has often been used [6] as the metric for a minimum distance classifier for texture classification. Within this work we utilise alternative metrics that take into account the statistical distribution of the test image as well as the training images. Figure 2 shows an example where the mean of two distributions can be equal but where the two distributions are obviously distinct. In order to distinguish these types of distribution, characterisation of one distribution is not enough. Therefore a Euclidean or Mahalanobis minimum distance classifier will be unable to distinguish these types of distributions. To overcome this problem two new metrics are introduced.

3.1. Divergence

Divergence is a measure of distance or dissimilarity between two classes based upon information theory [8]. An expression for the (average) discriminating information between

two feature populations can be calculated from the populations' entropy. From this expression a measure of divergence can be derived. In a multivariate case this expression for the divergence is given by:

$$D_{ij} = \text{tr} [(C_i - C_j) (C_j^{-1} - C_i^{-1})] + \delta^T (C_j^{-1} - C_i^{-1}) \delta \quad (1)$$

Where C_i and C_j are the covariances of the texture classes i and j respectively and δ is the difference in the class means represented as

$$\delta = m_j - m_i \quad (2)$$

where tr is the matrix trace function.

3.2. Bhattacharya Distance

The divergence measure of distinguishing two class populations is formed by considering class entropies. An alternative measure can be formed by estimating the probability of error as an upper bound on the Bayes error for normally distributed classes [9]. This measure is the Bhattacharya distance and is defined as:

$$B_{ij} = \frac{1}{2} \ln \frac{|C_i + C_j|}{|C_i|^{1/2} |C_j|^{1/2}} + \frac{1}{8} \delta^T \left(\frac{C_i + C_j}{2} \right)^{-1} \delta \quad (3)$$

The Bhattacharya metric has already been used for characterising wavelet decomposed images [10]. However, no inter-subband correlation was modelled within this work, as the metric was only used to obtain a measure between colour distributions within each wavelet subband.

Although these two metrics are both formulated to measure the differences between two multivariate distributions they are both included for potential differences in computational and classification efficiency.

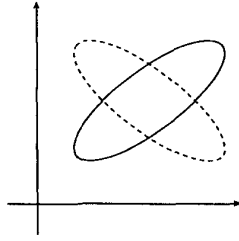


Fig. 2. Two 2D distributions sharing the same mean value

4. STATISTICAL MODELLING OF SUBBAND COEFFICIENTS

Many different parametric models have been suggested for generalised probability distribution functions of wavelet subband coefficients [3, 11, 12]. We adopt the form given by Birney and Fischer [11], and Tanabe and Farvardin [12]. This form states that subband coefficients obey the probability distribution:

$$p(x) \propto (-[\eta(\nu, \sigma)]) \quad (4)$$

where

$$\eta(\nu, \sigma) = \frac{1}{\sigma} \left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)} \right]^{\frac{1}{2}} \quad (5)$$

and Γ is the well known "gamma" function. ν is the governing parameter within this equation. $\nu = 1.0$ yields a Laplacian density function whilst $\nu = 2.0$ yields a Gaussian density function. Most sources [3, 11, 12] have concluded that for natural images, wavelet coefficients have a probability density function (pdf) defined by the parameter value ν equal to unity (i.e. Laplacian) or slightly less than unity (0.7 is suggested by Birney and Fischer). These are the distributions of the actual coefficients where the values can be positive and negative. Within this work we are interested in the distribution of the averaged (positive) energy values within each subband as described above. As the divergence and Bhattacharya metrics introduced in section 3 work on the assumption of Gaussian distributions, the implication for texture classification is that these classifiers will under-perform if the energy measures are significantly non-Gaussian. It is therefore important to check whether the distributions of the averaged energy values are Gaussian.

4.1. Estimation of Probability Density Function Parameters

In order to further test the above theories the ν parameter was estimated for each of the ten features extracted from

each wavelet subband for all of the same (112) texture images. ν was estimated using equations 6 and 7 taken from [3].

$$\nu = F^{-1} \left(\frac{E[|X|]}{\sigma} \right) \quad (6)$$

$$F(\alpha) = \frac{\Gamma(2/\alpha)}{\sqrt{\Gamma(1/\alpha)\Gamma(1/\alpha)}} \quad (7)$$

As gamma is not an easily invertible function a 10 000 point look up table was used to obtain estimates of ν from the variance and expectation of the absolute mean.

The results (the mean of each ν for each subband for all 112 textures) are shown in table 1. These results are for the average energy measures extracted from each imaginary region in each subband and not the individual coefficient values. All the subbands exhibit subband distributions close to Gaussian (i.e. $\nu \approx 2.0$).

Subband (Figure 3)	Estimated Probability Parameter ν
0	2.478824
1	1.759730
2	1.737486
3	1.675581
4	1.775865
5	1.825954
6	1.687149
7	1.845527
8	1.983405
9	1.848081

Table 1. Generalised distribution parameter (ν) for imaginary region averaged energy values

0	1	4	7
2	3		
5	6		
8		9	

Fig. 3. Wavelet subband index

5. RESULTS

Table 2 shows the results of using the four distance metrics: Euclidean, Mahalanobis, Bhattacharya and divergence

within the experiments described in section 2.1. The Euclidean metric performs very poorly but provides a baseline to compare the other metrics that take into account feature distributions. The Mahalanobis metric improves considerably on the Euclidean result by considering the feature distribution within the training textures. A further improvement in classification rate was achieved by the Bhattacharya and divergence metrics. This is as one would predict as these metrics consider both the feature distribution of the test and training features.

Metric	Correct Classification Rate (%)
Euclidean	43.97
Mahalanobis	59.86
Divergence	69.61
Bhattacharya	70.16

Table 2. Classification rates for four distance metrics

6. CONCLUSION

In this paper we have attempted to develop a suitable statistical model for the statistics of a typical wavelet decomposition. Subsequently, suitable distance metrics were identified for this distribution. A novel structure for extracting multivariate statistics from a wavelet decomposition was used in conjunction with these metrics to produce an improved texture classification method.

The results of the experiments could be considered to be inferior to many similar texture classification experiments within the present subject literature. However, this is not a realistic comparison as the above experiments used only one texture for training and attempted to classify each test texture as being one from the entire set of 112 Brodatz textures. This is a considerably more difficult task than most previous texture classification experiments that use many training textures and a much smaller subset of the Brodatz images. The relative increase of classification rates of the divergence and Bhattacharya metrics over the Mahalanobis and Euclidean metrics indicates a potential increase in accuracy of a image retrieval application that uses such a method for subband statistical analysis.

This method could as easily be applied to the subbands of many other wavelet decompositions (e.g. complex wavelets, wavelet packets, steerable pyramid etc.). Further results have been produced for variously sized "imaginary regions". These results were not significantly different.

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